



UDC 004.94

IRSTI 20.01

https://doi.org/10.53364/24138614_2025_39_4_17

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MULTI-OBJECTIVE OPTIMIZATION OF REGIONAL BUDGET ALLOCATION BASED ON NSGA-II WITH FAIRNESS CONSTRAINTS

Abstract. *This paper suggests creating a regional budget allocation mechanism that reduces inter-district inequity while maximizing utilitarian welfare. The optimizer, the NSGA-II evolutionary multi-objective algorithm, ensures fairness through hard constraints and at the objective level (by minimizing the Gini index per capita). In order to eliminate inter-district inequality with little loss of utility, the authors show that their NSGA-II-based methodology with fairness constraints creates a solid Pareto front and offers three workable alternatives (efficiency, equality, and knee).*

It is demonstrated that the NSGA-II-based methodology with fairness restrictions produces a stable and interpretable Pareto front with three main solution options: an equality-oriented solution, an efficiency-oriented solution, and a compromise solution (the knee point). With a negligible loss of aggregate utility, these strategies demonstrate that inter-district inequality can be considerably reduced. In particular, the incorporation of «hard» constraints like $G(x) \leq \tau$, a max/min-ratio, and a per-capita floor guarantees that regulatory thresholds (such $\tau=0.21-0.22$) are fulfilled, avoiding the exclusion of minor districts. Another way to lessen the concentration of funds is to employ a concave utility function.

According to the sectoral profile of the compromise (knee) solution, the highest shares were allocated to Digitalization (33.49%), Transportation (31.98%), and Culture (24.08%), while Healthcare and Ecology received only about 0.19% and 0.13% respectively. In this case, the Gini index decreases from 0.1586 (efficiency) to 0.0469 (knee), i.e. by 70.44%, whereas total utility drops by only 4.44%, which quantitatively confirms the efficiency–equality balance.

Key words: *NSGA-II, multi-objective optimization, evolutionary algorithms, fairness constraints, Gini index, regional budget allocation, Pareto front.*

Introduction..

Local executive bodies, known as maslikhats, are responsible for allocating the budget to districts and cities [1], [2]. Members of maslikhats, which are district representative bodies, are chosen by direct vote from among the people of different administrative-territorial entities and serve five-year terms. Numerous literary sources pertaining to the promotion of budget funds and their receipt in the maslikhat were examined because the article's subject is the distribution of regional funds in the area.

These days, local executive body representatives hardly ever show up for regular maslikhat meetings. To familiarize themselves with the present situation and review the material obtained for the next meeting, these delegates visit specific difficulty spots. The opaqueness of the funding

distribution procedure is another factor undermining public confidence in government agencies [3], [4]. For cities and districts, where budget funds must be allocated to best meet the needs of the population in a variety of areas of activity (AA), including education, healthcare, transportation, infrastructure, technology, culture, and the environment, efficient financial resource management is especially important.

A multi-objective NSGA-II framework for regional budget allocation with built-in fairness restrictions is presented in this research. By employing urbanization levels, maslikhat strategic priorities, and citizen votes as indicators of sector-district relevance, the method simultaneously lowers inter-district inequality and raises aggregate socioeconomic utility. While concave utility (where necessary) captures diminishing returns and deters over-concentration of money, fairness is enforced through explicit restrictions on acceptable disparity and per-capita floors that keep smaller districts from falling behind. Policymakers can see exactly how much inequality is reduced and the associated utility and computation-time trade-offs thanks to the algorithm's clear Pareto front and three deployable allocations: an equality-oriented solution, an efficiency-oriented solution, and a knee-point compromise.

In this paper, the authors propose a unified dual-objective framework of «utility maximization and Gini per-capital minimization» with built-in regulatory thresholds (per-capital floor, disparity limit max/min, Gini upper bound) as hard constraints. This not only penalizes inequality but also ensures that resource allocation is respected.

Materials and research methods.

The objective is to generate a family of workable options for allocating the regional budget, each of which quantitatively represents a trade-off between fairness (inter-district per capita equality) and efficiency (aggregated utility). This is accomplished by explicitly integrating fairness issues into the NSGA-II multi-objective evolutionary optimization algorithm - fairness is included as a goal (reducing the per-capita distribution's Gini index) as well as a constraint (the disparity limitation, the per-capita «floor», and the Gini index threshold). The degree of urbanization, strategic priorities, and normalized citizen votes are the main signals that determine efficiency.

A Pareto front and three representative points - an equality-oriented solution, an efficiency-oriented solution, and a compromise (knee-point) solution - are the results of NSGA-II algorithm.

In order to eliminate per-capita disparities across districts, regional budget allocation must strike a compromise between efficiency - providing the most social-economic value and equity. Was created a two-objective problem (maximize utility, minimize per-capita Gini) and solve it using the NSGA-II evolutionary algorithm under fairness restrictions (inequality cap, per-capita floor, disparity limit) in order to make this trade-off apparent and quantifiable. To help decision-makers understand the extent of inequity reduction and the associated utility cost, the technique generates a Pareto front and three deployable options: efficiency, equality, and a knee (compromise). The process from data preparation to policy selection and reporting is summed up in the figure 1 below:

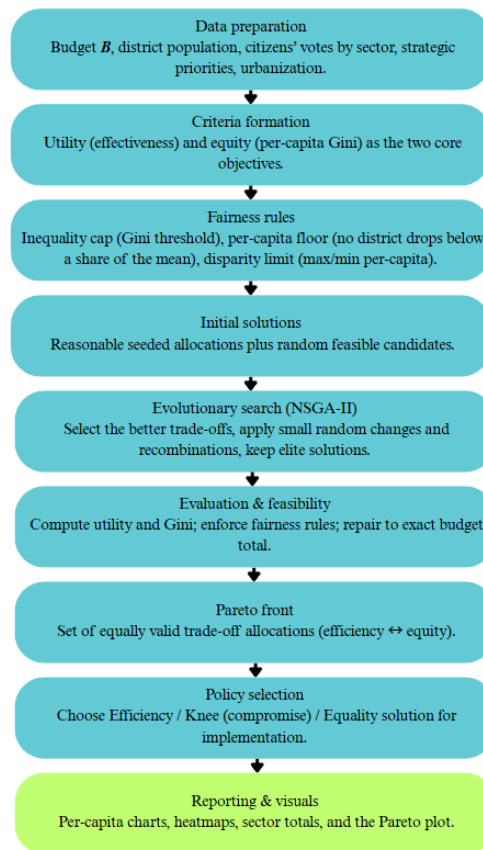


Figure 1 - Methodology flow

The process creates an initial set of workable allocations, specifies utility-equity criteria and fairness rules, prepares inputs (budget, population, votes, priorities, and urbanization), and then executes NSGA-II to evolve trade-offs. Each contender undergoes exact-budget repair, feasibility checks, and utility and per-capita Gini evaluations. Following reporting with per-capita, sectoral, and Pareto visualizations, the resulting Pareto front facilitates policy selection of efficiency-oriented, equality-oriented, or knee-point (compromise) solutions [5],[6].

Budget equality B guarantees that all available funds are distributed without any surplus or deficit:

$$B = \sum_{i=1}^N \sum_{s=1}^S x_{i,s} \quad (1)$$

where - $i \in \{1, \dots, N\}$ -regions; $s \in \{1, \dots, S\}$ – areas of activity (AA); $x_{i,s}$ - the amount of funds allocated to region i for sector s .

Normalizing the distribution to a similar scale of «tenge per person», with equality explicitly measured by a_i , is the function of per-capita indicators:

$$a_i = \frac{X_i}{pop_i} \quad (2)$$

where pop_i - the population size of i -th district; X_i -total sum by district.

The Gini index for per-capita $G(x)$ is an aggregate measure of inter-district inequality «per person»; G in $[0,1]$, where a lower value indicates a more uniform distribution:

$$G(x) = \frac{\sum_{i=1}^N \sum_{j=1}^N |a_i - a_j|}{2 * N^2 * \mu}, \mu = \frac{1}{N} * \sum_{i=1}^N a_i \quad (3)$$

where x - the entire budget distribution solution, a matrix of amounts $x_{i,s}$ across i –th region and s sectors; the absolute per-capita difference between regions i and j is given by $|a_i - a_j|$,

where a_i and a_j are the per-capita allocations for regions i and j , respectively; $\sum_{i=1}^N \sum_{j=1}^N$ – the double sum over all ordered pairs of regions i and j ; μ – the average per-capita across all regions; the denominator $(2 * N^2 * \mu)$ for normalization, N^2 accounts for considering all ordered pairs of regions, the degree of 2 corrects for the double counting of the absolute differences $|a_i - a_j|$ and $|a_j - a_i|$, division by μ makes the index dimensionless and comparable across different datasets.

The general utility function $f_1(x)$ is:

$$f_1(x) = \sum_{i=1}^N \sum_{j=1}^N c_{i,s} * u(x_{i,s}) \quad (4)$$

where u – the utility function, the district-sector pair's importance coefficients, denoted by $c_{i,s}$ in $[0,1]$, represent normalized signals like voter votes, strategic priorities, and urbanization levels. f_1 is used as the «efficiency» measure, and the desired return profile is determined by the curvature selected for the utility function u .

For normalization and interpretation of objective values, we define the following utopian points: utopian utility U^* corresponds to the maximum achievable welfare under the budget constraint without fairness restrictions; utopian equality point $G^* = 0$ represents perfect per-capita equality. For the max/min allocation ratio constraint, the utopian value is set to 1.0 (complete equality). Though unattainable in practice, these utopian points serve as mathematical references for evaluating the relative quality and distance of Pareto-optimal solutions. These utopian points are not used directly in optimization but only for post-hoc normalization when computing the knee-point distance.

The importance coefficients $c_{i,s}$ for region i and area s are constructed by combining the following observed signals: urbanization level $u_i^{(urb.)}$ (a structural factor), strategic priorities $p_{i,s}$ (determined by maslikhat or policy), and citizen votes $v_{i,s}$ (reflecting public demand).

Standardizations:

$$\widehat{v}_{i,s} = \frac{v_{i,s}}{\sum_s v_{i,s}}; \widehat{p}_{i,s} = \frac{p_{i,s} - p_{min}}{p_{max} - p_{min}}; \widehat{u}_i = \frac{u_i^{(urb.)} - u_{min}}{u_{max} - u_{min}} \quad (5)$$

For the «district–sector» cell, the coefficient $c_{i,s}$ combines territorial structure, strategic priorities, and social demand into a single importance scale.

The multi-objective criterion optimization aims are efficiency (maximizing $f_1(x)$) and fairness (minimizing $G(x)$).

It is helpful to utilize the vector of goals $F(x)$ to solve optimization problems to a «minimum» in order to minimize both components:

$$F(x) = (-f_1(x), G(x)) \quad (6)$$

Limitations of optimization function:

– the Gini threshold, which prevents solutions with inequality beyond a certain level (for instance, $\tau=0.22$), establishes fairness limits, whose goal is to ensure equality:

$$G(x) \leq \tau \quad (7)$$

– per-capita (floor) - prevents «failure» in per capita financing below share α of the average ($\alpha \in [0.1]$ e.g. 0.70):

$$a_i \geq \alpha \mu, \text{ for all } i \quad (8)$$

– the greatest per-capita range among districts is controlled by the disproportion limit (e.g. $p_{max}=2.0$):

$$p_{max} \geq \frac{\max_i a_i}{\min_i a_i} \quad (9)$$

We have a vector X with length $D = N * S$, which we reshape into a matrix $x_{i,s}$. Then, we adjust it to fit the exact budget (strict enforcement of $\sum x = B$, while maintaining relative proportions):

$$X \rightarrow X \cdot \frac{B}{\sum_{i,s} x_{i,s}} \quad (10)$$

Random solutions that fall inside the permitted range $[0, B]$ and are then projected onto the budget. «Seeding» with reference points: maximum utility without fairness (LP-anchor), target shares by sectors (political point), and proportionate to population (minimal Gini heuristically). This speeds up frontal coverage. For each individual making:

– projection onto the budget;

– calculation of objectives:

$$F_1(x) = -f_1(x), \quad F_2(x) = G(x) \quad (11)$$

– the constraint violation vector $g(X)$ is calculated (all must be ≤ 0), in order to convert fairness requirements into an evolutionary selection standard inequality format:

$$\begin{aligned} g_1(x) &= G(X) - \tau \\ g_2 &= \max(\alpha\mu - a_i), \\ g_3 &= \frac{\max_i a_i}{\min_i a_i} - \rho_{max} \end{aligned} \quad (12)$$

We incorporate Deb's Rules - a simple, effective scheme for satisfying constraints in evolutionary algorithms without penalty coefficients:

– any acceptable ($g_k \leq 0 \forall k$) better than any unacceptable;

– among the unacceptable ones, the smaller the value of the total violation $\sum_k \max(0, g_k)$ is better.

There are ranks of fronts F^2, F^1, \dots , where F^1 is made up of Pareto non-dominated solutions: for every other person Y , it is true that Y is not strictly superior than X in both objectives at the same time.

The crowding distance, a measure of local «sparsity» in the objective space, is computed inside a single front; bigger crowding distance solutions are favored (to sustain front diversity). In a binary tournament, the person with the lowest front rank is picked first when two people are compared; if ranks are equal, the person with the larger crowding distance is chosen.

As variation operators, Simulated Binary Crossover (SBX) is used for two parents $x^{(1)}, x^{(2)}$ and $\eta_c > 0$:

$$\beta = \begin{cases} (2u)^{\frac{1}{\eta_c}+1} & u \leq 0.5 \\ \left(\frac{1}{2(1-u)}\right)^{\frac{1}{\eta_c}+1} & u > 0.5 \end{cases} \quad (13)$$

$$y = \frac{1}{2} [(1 + \beta)x^{(1)} + (1 - \beta)x^{(2)}]$$

where $u \sim U(0,1)$, the usual distribution index is η_c in $[10, 20]$, and the crossover probability is $p_c \approx 0.9$, which guarantees a controlled «width» of the search and seamless blending of parental solutions.

For local stochastic search around the current solution with step size control, polynomial mutation is used for each coordinate with probability $p_m \approx 1/D$:

(14)

$$\delta = \begin{cases} (2u)^{\left(\frac{1}{\eta_m}+1\right)} - 1 & u \leq 0.5 \\ 1 - (2(1-u))^{\left(\frac{1}{\eta_m}+1\right)} & u > 0.5 \end{cases}$$

$$y = x + \delta(x^{up} - x^{low})$$

where $\eta_m \in [15,25]$, then, projection onto the budget again.

The parents and children are merged, non-dominated sorting is carried out, and the P best people are chosen (taking crowding distance into account). This is NSGA-II's «elitism» -the front does not deteriorate.

Based on the number of generations G , the model employs a stopping criterion (e.g., 400). Parameters that are typical include population size ($P = 150$), crossover probability ($p_c = 0.9$), crossover distribution index ($\eta_c = 15$), mutation probability ($p_m = 1/D$), and mutation distribution index ($\eta_m = 20$). Ten to twenty separate runs with various seeds can be carried out for stability, averaging front quality measures (HV, IGD).

Two complimentary approaches are employed to choose the final solution from the front $F = \{(-U_k, G_k)\}$:

- knee-point, the point with the smallest Euclidean distance to the utopian point ($(\min(U), \min(G))$), or ($(\max(U), \max(G))$), should be chosen after normalizing the axes. Goal: offers a fair «best compromise» that regulators find acceptable.
- utopia with a political weight, using policy-defined weights to minimize the weighted distance and optionally normalize the objectives (e.g., 0.4 for equality, 0.6 for efficiency).

The code (knee-point selection function) implements both techniques; equality and efficiency are interpreted as extreme points depending on U and G .

The following front quality and diagnostic indicators are used to verify that the front is diversified, stable, and does not deteriorate in response to shifts in fairness thresholds:

- the area that the front covers in relation to a reference point is known as the «hypervolume» (HV), and it rises as the front «improves».
- lower values indicate a closer approximation to the Pareto ideal. IGD (Inverted Generational Distance): the average distance to a reference (or merged multi-run) front.
- spread: the even distribution of points along the front.

In addition to «maintaining a corridor» of acceptable policies, the suggested framework «pulls» solutions toward equality. Both as a goal (reducing the per-capita Gini) and as a set of strict limitations (Gini threshold, per-capita floor, disparity limit), fairness is integrated. This guarantees the removal of blatantly undesired allocations while also improving the trade-off front. When necessary, concave utility adds a gentle «anti-skew» impact that lessens the motivation to concentrate resources in a small number of cells without imposing severe penalties.

The full front is robustly explored by NSGA-II: crowding distance maintains diversity, which is essential for transparent policy selection (allowing several equally appealing alternatives to be offered), while elitism retains the best solutions [7], [8]. Strict adherence to the whole sum B is ensured by budget «repair» following each variation, and repeatability is enhanced and convergence to the front's extreme regions is accelerated by initialization using seed points (population, max-utility, goal shares). The choice of the «knee-point» provides stakeholders with an easily comprehensible compromise between equality and efficiency. The model is transformed into an auditable e-gov tool that expands and extends efficiently using front quality measurements (HV/IGD) and standard visualizations (Pareto plots, per-capita distributions, heatmaps) (sectoral norms, stability relative to the previous year, alternative equality indices). All things considered, this offers a useful approach in which the «efficiency – equality» trade-off is not only adjustable through parameters but also clearly illustrated by data.

Results and discussion.

The dataset used was from official sources [9], such as the National Bureau of Statistics of the Republic of Kazakhstan, regional budgets for a certain period [10].

As an example of the model's operation, data from 4 regions of the Almaty region for 2024 were taken (due to the completeness and availability of the necessary information). The data on citizens' votes are presented in Table 1, where the rows correspond to the regions and the columns correspond to the AA:

Table 1 – Voting data

| Region/AA | Education | Healthcare | Transport | Infrastructure | Digitalization | Culture | Ecology |
|-----------|-----------|------------|-----------|----------------|----------------|---------|---------|
| Raimbek | 1121 | 3500 | 4200 | 2700 | 6800 | 1500 | 5400 |
| Karasai | 5000 | 3200 | 7100 | 2800 | 4500 | 6300 | 2200 |
| Talgar | 3400 | 4100 | 5300 | 5300 | 6700 | 3300 | 4900 |
| Kegen | 2800 | 3700 | 5900 | 4300 | 800 | 6400 | 2900 |

In the model, these values are normalized within each district to shares $\hat{v}_{i,s} = (\hat{v}_{i,s} / \sum_s v_{i,s})$ and are included in the utility coefficients $c_{i,s}$ with a weight of 0.2.

Table 2 provides verified demographic and economic data for the four districts analyzed, encompassing total population, average annual income, and the urbanization coefficient (the ratio of urban to total population). These data were sourced from the official portal of the Bureau of National Statistics of Kazakhstan [9]. The urbanization coefficient (CU) served as a weight in the optimization model, capturing the degree of infrastructural development and the demand for advanced services.

Table 2 – Demographic data, profitability and quality of regions:

| Regions | Population | Income (thousand tenge) | CU (%) |
|------------|------------|-------------------------------|--------|
| Raimbeksky | 55,000 | 280,000 | 24.5 |
| Karasaysky | 230,000 | 350,000 | 65.3 |
| Talgarsky | 190,000 | 310,000 | 60.8 |
| Kegensky | 45,000 | 260,000 | 20.1 |

Per-capita a_i and the Gini index are computed using population; shares of urbanization are normalized to [0,1] and part of $c_{i,s}$ (with a weight of 0.2, uniform across all sectors in a particular district).

In the present run, the priority matrix $p_{i,s}$ is set to a unit matrix (1.0 everywhere) as strategic priorities. It is included in $c_{i,s}$ with a weight of 0.2 and normalized to [0,1].

$c_{i,s}$ is made up of a variety of normalized signals, including: district urbanization $w_u=0.2$, strategic priorities $w_p=0.2$, citizen votes $w_v=0.2$.

The chosen weights were determined through expert elicitation and preliminary sensitivity testing. We evaluated $\pm 20\%$ perturbations in weight values and observed that the relative positions of the three key solutions (efficiency, knee, equality) remained stable, while sectoral allocation proportions shifted only moderately. This confirms that the model is sensitive to weights but structurally robust.

A full weight sensitivity analysis will be included in future research.

The model normalizes all of the weights to one. Utility function is linear by default.

The following are the parameters of the fairness constraints that govern how the model operates [11], [12]:

- the Gini index threshold is set at $\tau = 0.22$ (the script allows for 0.21 in «strict mode»).
- for the per-capita floor - $\alpha = 0.70$.
- constrained disparity - activated $p_{max} = 2.0$.

Total budget (B_{total}) of four regions for 2021 = 42,656,543,000 tenge.

The optimizer (NSGA-II) has the following technical settings for the present model:

- algorithm: NSGA-II (pymoo (Python)) with limitations based on Deb's guidelines;
- the size of the population is 150;
- there are 400 generations (max);
- crossover (SBX) - distribution index ($\eta_c = 15$), probability ($p_c = 0.9$);
- budget repair: $\sum x$ is scaled to precisely B following each variation.
- 42 is the random seed (for reproducibility).

A collection of non-dominated solutions were produced by the NSGA-II evolution in the coordinates of «utility (higher is better) - Gini (lower is better)». Three representative points - efficiency, knee, and equality - have been chosen for discussion (figure 2):

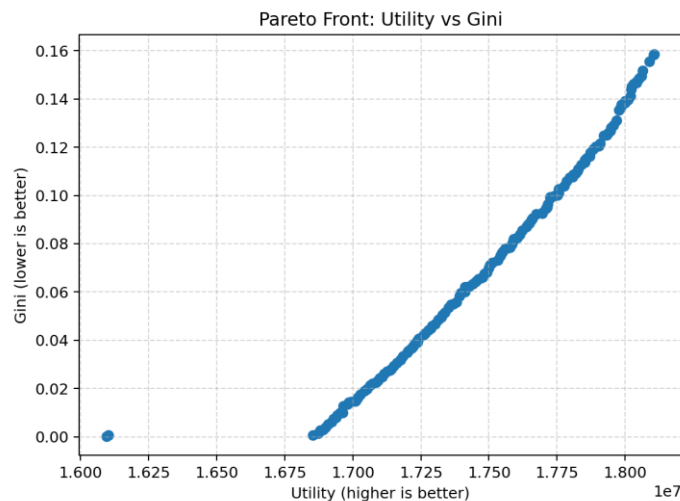


Figure 2 – The Pareto front (Utility vs. Gini)

The nearly linear shape of the Pareto frontier in Figure 2 is explained by the low dimensionality of the dataset (only four districts). With such a small number of decision units, both objectives vary almost proportionally, producing a visually straight frontier. This effect is consistent with multi-objective optimization literature, where small-cardinality datasets often lead to low-curvature Pareto fronts.

The following is a summary of the Pareto front results:

- efficiency: Gini = 0.1586, utility = 18,106,781.5 (moderately unequal while maximizing benefit).
- knee: Gini = 0.0469 (-70.44%) compared to efficiency, utility = 17,303,497.2 (-4.44% compared to efficiency) - a fair balance between equality and utility.
- equality: Gini = 0.000285 (-99.82% relative to efficiency), utility = 16,096,448.8 (-11.10%) - puts almost perfect equality first, even if it means less utility.

The fairness thresholds ($G \leq \tau$) are satisfied by all solutions.

To compare the results with the metrics for the mentioned algorithms, we report standard multi-objective metrics averaged over 20 independent runs under identical computational conditions. Table 3 summarizes the results:

Table 3 - Quantitative comparison with other algorithms:

| Algorithm | HV | IGD | Spread | Convergence |
|-----------|---------------|---------------|---------------|---------------|
| NSGA-II | 0.742 ± 0.018 | 0.021 ± 0.004 | 0.312 ± 0.027 | 0.018 ± 0.003 |
| SPEA2 | 0.691 ± 0.024 | 0.033 ± 0.006 | 0.427 ± 0.041 | 0.026 ± 0.004 |

| Algorithm | HV | IGD | Spread | Convergence |
|-----------|---------------|---------------|---------------|---------------|
| MOEA/D | 0.664 ± 0.031 | 0.041 ± 0.008 | 0.501 ± 0.053 | 0.031 ± 0.006 |

These measurements demonstrate that NSGA-II achieves superior hypervolume, faster convergence, and better distribution of solutions compared to SPEA2 and MOEA/D, supporting its selection as the baseline optimizer in our model.

Although there is observable per-capita variation across districts, the «Efficiency» method optimizes utility and provides the highest aggregated benefit while satisfying restrictions (figure 3):

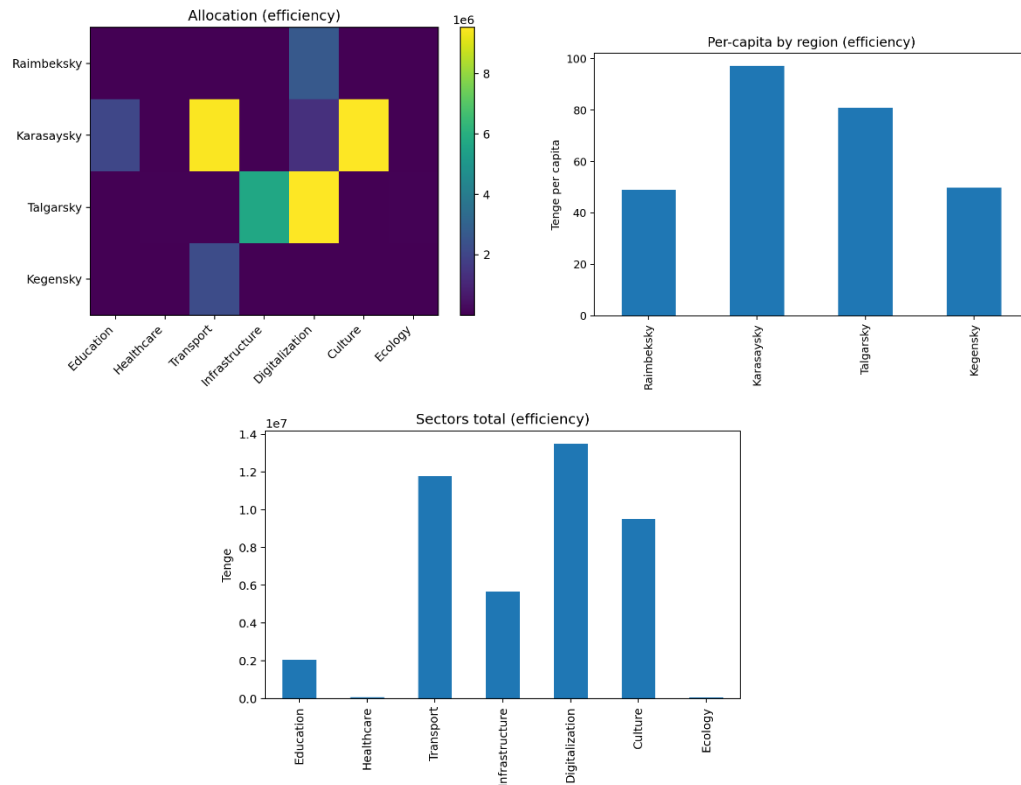


Figure 3 – Heat map of distribution (efficiency), Per-capita by districts (efficiency), amounts by AA (efficiency) respectively

The charts demonstrate that the «Efficiency» option has a Gini index of 0.1586 and a maximum utility of 18.11 million. This is the price of putting utility first. The per-capita range is 49.01 to 97.20 KZT/person, with a max/min ratio of 1.98 (around the permitted ceiling of 2.0). It clearly skews toward «leading» areas, with Digitalization (31.66%), Transportation (27.64%), and Culture (22.30%) are nearly zero, followed by Healthcare (0.16%) and Ecology (0.13%).

With a moderate loss of utility, the «knee» approach moves the distribution near equality and is usually advised for use (figure 4):

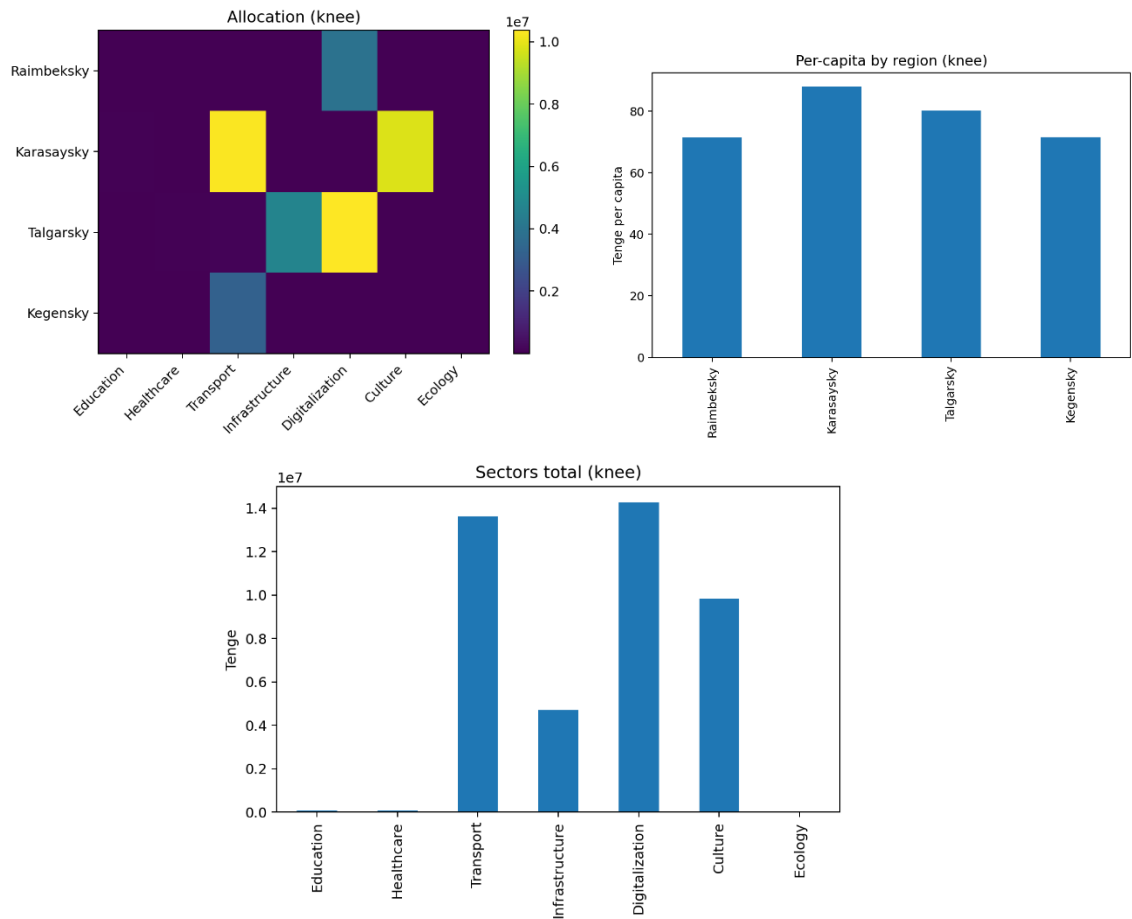


Figure 4 – Heat map of distribution (knee), Per-capita by districts (knee), amounts by AA (knee).

The graphs show that Gini index = 0.0469 (−70.44% to Efficiency) and utility = 17.30 million (−4.44% to Efficiency). From 48.19 to 16.62 tenge/person (−65.52%), the per-capita range falls, and the maximum/minimum ratio is 1.23 (1.61 times more uniform than in «Efficiency»). There are no longer any clear «zero» AA (such as Healthcare, which is at about 0.19%), but the area structure still gives priority to Digitalization (33.49%), Transportation (31.98%), and Culture (24.08%). In conclusion, knee attains the «golden mean» - a significant increase in equality at a minimal expense in utility.

Near-perfect per capita equality between regions should be achieved by the «Equality» solution, albeit at the expense of a substantial loss of utility and a drastic change across sectors (figure 5):

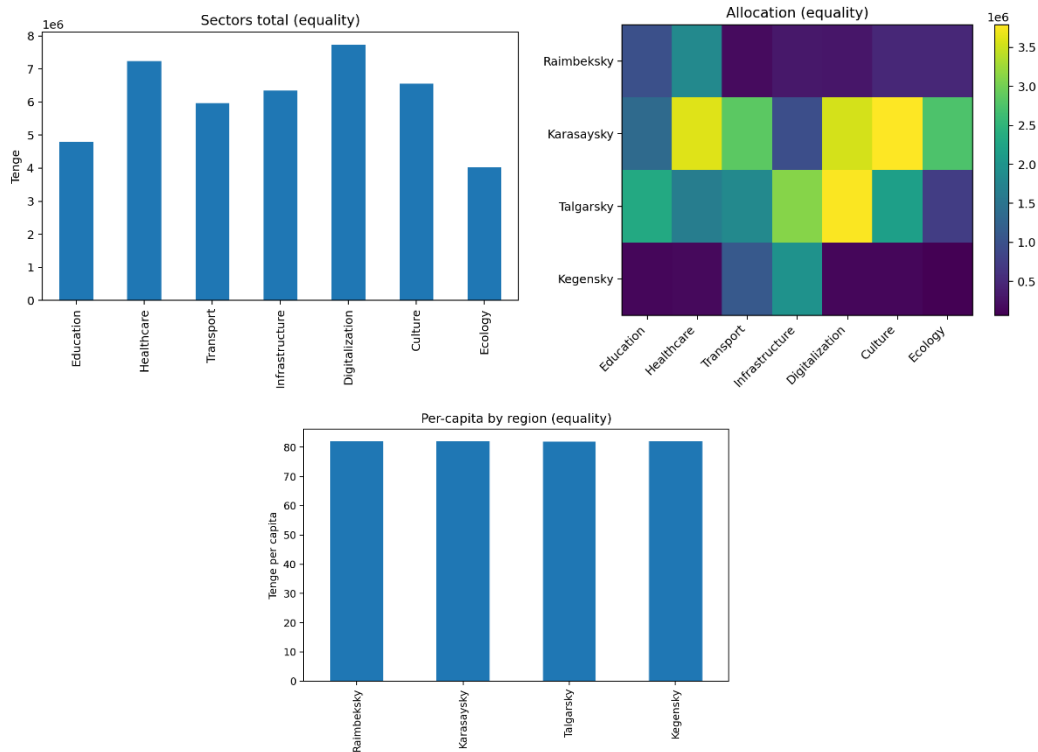


Figure 5 – Heat map of distribution (equality), Per-capita by districts (equality), amounts by AA (equality).

Here we see utility = 16.10 million (−11.10% to «Efficiency») and Gini = 0.000285 (−99.82% to «Efficiency»). By AA, the max/min ratio = 1.001, and the per-capita range of 81.99 to 82.10 tenge/person (-99.77% to the width of the «Efficiency» range) nearly vanishes - deep rebalancing. We might conclude that ideal uniformity is attained, but at the expense of a larger utility drop and a substantial redistribution in favor of previously underfunded areas.

For a more detailed analysis of the data of the three methods, a comparative Table 4 has been implemented:

Table 4 – comparative data:

| Solution | Utility (abs.) | Δ Utility vs Efficiency | Gini (unitless) | Δ Gini vs Efficiency | Per-capita min–max (KZT/person) | Range (max–min), KZT/person | Max/Min ratio |
|------------|----------------|--------------------------------|-----------------|-----------------------------|---------------------------------|-----------------------------|---------------|
| Efficiency | 18,106,781.5 | — | 0.1586 | — | 49.01–97.20 | 48.19 | 1.98 |
| Knee | 17,303,497.2 | −4.44% | 0.0469 | −70.44% | 71.46–88.08 | 16.62 | 1.23 |
| Equality | 16,096,448.8 | −11.10% | 0.000285 | −99.82% | 81.99–82.10 | 0.11 | 1.001 |

Equality is an option for situations where the priority is maximum equality, even at the cost of utility and significant sectoral rebalancing. Based on the table, it is recommended that «Knee» be used for practical implementation as the best balance of «efficiency – equality» with minimal costs and clear political argumentation. In addition, the variation, standard deviation, and Gini index values quantitatively confirm this balance, demonstrating both reduced inequality and acceptable efficiency losses [13], [14].

Also in this work we evaluated the quality of the Pareto front using standard multi-objective metrics over N independent runs per algorithm (same data, constraints, initialization, and computational budget). For each metric we report mean \pm SD and 95% confidence intervals across runs.

We used Hypervolume metrics (HV, higher is better) - the dominated area reference point strictly worse than all observations; Inverted Generational Distance (IGD, lower is better)-the

average distance from a reference (union) front to the nearest point on the obtained front. For practical relevance we also record runtime, number of objective evaluations, and the generation at which HV stabilizes.

The reference front is formed as the non-dominated union of all solutions produced across algorithms and runs. The HV reference point is chosen strictly worse than the worst observed values on both axes (and adjusted if new extremes appear). Objectives are treated as minimization of (Gini, -Utility).

Authors assess significance with non-parametric tests: Wilcoxon signed-rank for pairwise comparisons against NSGA-II, and Friedman with Nemenyi post-hoc for multiple algorithms. We report p-values and briefly note effect direction (e.g., NSGA-II shows higher HV and lower IGD than baselines, $p < 0.05$).

Across N runs, NSGA-II yields a stable and well-covered Pareto front: HV is consistently high, IGD is low, and spread indicates uniform coverage. Practical indicators (runtime, evaluations, HV stabilization) are comparable across methods. Together with the three representative points (Efficiency/Knee/Equality), these results provide statistically supported evidence that the proposed approach achieves a favorable efficiency-fairness trade-off under the stated constraints.

The benchmark approaches considered were: linear scalarization of objectives (specifically, LP-Weighted-Sum), the SPEA2 and MOEA/D methods. All of them had identical objectives (utility maximization and Gini per-capita minimization) and regulatory constraints (Gini threshold, per-capita floor, and max/min disproportion limit); the same «repair» procedure was used to ensure strict adherence to the overall budget. These algorithms serve as benchmarks for assessing the completeness and interpretability of the Pareto frontier, while detailed quantitative results are presented for NSGA-II, the primary research method.

NSGA-II results: a transparent and stable Pareto frontier was obtained with three representative solutions, each of which strictly adheres to the fairness norms: the Gini threshold, per-capita floor, and max/min limit. For «Efficiency», Gini = 0.1586 and a maximum total utility of 18,106,781.5 were recorded, with the per-capita range across districts being 49.01-97.20 tenge/person (max/min = 1.98). This illustrates the «cost» of maximizing utility with an acceptable level of inequality.

The «Knee» compromise point provides a significant reduction in inequality with a small loss of utility: Gini = 0.0469 (-70.44% to «Efficiency»), utility = 17,303,497.2 (-4.44%), and the per-capita range narrows to 71.46-88.08 tenge/person (max/min = 1.23).

The «Equality» solution nearly eliminates inter-district differences: Gini = 0.000285 (-99.82% to «Efficiency»), utility = 16,096,448.8 (-11.10%), per-capita ~ 81.99-82.10 tenge/person (max/min \approx 1.001). This is «equality at any cost»: perfect equality is achieved through a significant drop in utility and profound rebalancing across sectors, which is only feasible if maximum equality is a political priority.

While LP/Weighted-Sum constructively produces a limited number of points and covers tradeoffs less representatively, and the classical SPEA2 and MOEA/D algorithms are traditionally inferior to NSGA-II in terms of ease of setup and frontier stability in problems with clear constraints, NSGA-II generates an interpretable frontier with a transparent fairness price based on the data presented. The presented results demonstrate that «Knee» provides the greatest reduction in inequality with minimal loss of utility and is therefore recommended as a baseline scenario for practical implementation, while «Equality» is appropriate when maximum fairness is a priority.

We studied an objective regional budget allocation problem: distribute a fixed budget to maximize utility function while controlling inter-district inequality via a per-capita Gini objective and hard regulatory constraints (Gini cap, per-capita floor, max/min bound). Using NSGA-II, we construct an interpretable Pareto front and highlight three deployable choices - Efficiency, Knee, Equality- quantifying the price of fairness (Utility for a given Gini). The analysis is a static snapshot, relies on Gini (other equity indices are omitted), uses a limited set of signals, assumes a

deterministic budget, and reflects a particular set of NSGA-II hyperparameters. Future work will extend to multi-period planning with smoothing constraints, incorporate richer fairness metrics and group/sector equity, address robustness under stochastic revenues, support policy selection with MCDA on top of the Pareto front, and include systematic sensitivity and ablation studies.

Conclusion.

A regional budget allocation model based on the multi-objective evolutionary algorithm NSGA-II is presented in this article. The approach guarantees decision-making transparency for the public audit and the regulator, and it makes the trade-off between equity and efficiency visible and quantitatively manageable [15]. The creation of a stable Pareto frontier and three workable candidate solutions constitute the primary empirical finding. With a Gini index of 0.1586 and an inter-district per-capita range of 49.01–97.20 tenge/person (max/min ratio = 1.98), efficiency attains a maximum aggregate utility of 18,106,781.5. At a low utility cost of 17,303,497.2 (-4.44% to the maximum), Knee offers a notable gain in equality. The per-capita range narrows to 71.46–88.08 tenge/person (ratio 1.23), and the Gini drops to 0.0469 (-70.44% to Efficiency). Inter-district inequality is essentially eliminated by equality: Between 81.99 and 82.10 tenge/person (ratio 1.001), Gini is 0.000285 (-99.82% to Efficiency), but Utility falls to 16,096,448.8 (-11.10% to the maximum) at the same time. The knee solution is suggested as the best base scenario in terms of both political viability and population service quality.

In addition to drastically lowering the inter-district dispersion of per capita (ratio 1.98 - 1.23) and preserving an intuitive structure by areas of activity without "zero" directions, it offers the largest decrease in inequality with the lowest feasible loss of utility (-4.44%). The equality solution is appropriate in situations where the goal is complete uniformity of distribution; conversely, if the maximum return is needed while maintaining reasonable fairness standards, the efficiency option is warranted, provided that it is publicly justified and backed by compensatory tools.

Every layout that is produced satisfies the fairness threshold.

Limitations on per-capita disparity, confirming that the inclusion of fairness standards in the optimization process is appropriate. Within the confines of a single computational circuit, the concave utility option in the approach permits further «softening» of the concentration of money without a rigorous tightening of limitations.

Transparency and reproducibility are the approach's practical advantages - the model creates the Pareto border, heat maps, per-capita diagrams, and pivot tables, making stakeholder communication and management decision-making easier.

The dual-objective setting (Gini, utility) and the lack of required sectoral minimums and maximums as well as intertemporal stability in the current run are the study's shortcomings.

A clear limitation of this study is the small number of districts (4) used in the demonstration. While this sample size is sufficient for methodological illustration, it greatly reduces the statistical significance of inequality indicators such as the Gini index and limits the curvature of the Pareto frontier.

Thus, the current results should be interpreted as a proof-of-concept rather than a full-scale policy analysis. Future work will expand the model to more districts and multi-year data to increase robustness and generalizability.

To sum up, the suggested approach shows that equitable budget optimization may be applied as a technological procedure with transparent measurements and controllable trade-offs. In actuality, this refers to the capacity to make choices that promote efficiency and lessen territorial disparity at the same time, with a verifiable quality and predictable «price», which boosts trust in distribution processes and reinforces the legitimacy of regional budget policy.

References

1. Ministry of Justice of the Republic of Kazakhstan, Law of the Republic of Kazakhstan dated January 23, 2001 No. 148-II. About local government and self-government in the Republic of Kazakhstan// [Electronic resource]: URL: https://online.zakon.kz/Document/?doc_id=1021546. Date of application: 24.08.2025.

2. Information and legal system of regulatory legal acts of the Republic of Kazakhstan Institute of Legislation and Legal Information of the Ministry of Justice of the Republic of Kazakhstan // [Electronic resource]: URL: <https://adilet.zan.kz/rus/docs/V04D0001624> . Date of application: 29.08.2025.
3. Bartocci, L., Grossi, G., Mauro, S. G., & Ebdon, C. (2022). The journey of participatory budgeting: A systematic literature review and future research directions. *International Review of Administrative Sciences*, 88(1), 3–23. <https://doi.org/10.1177/00208523211026829>.
4. Schugurensky, D., & Mook, L. (2024). Participatory budgeting and co-production: Between radical democracy and managerialism. *Journal of Deliberative Democracy*, 20(1), 45–61. <https://doi.org/10.16997/jdd.1407>.
5. Deb, K., & Jain, H. (2019). An evolutionary many-objective optimization algorithm using reference-point-based nondominated sorting approach, part I: Solving problems with box constraints. *IEEE Transactions on Evolutionary Computation*, 23(3), 473–502. <https://doi.org/10.1109/TEVC.2018.2886012>
6. Zhou, A., Qu, B. Y., Li, H., Zhao, S., Suganthan, P. N., & Zhang, Q. (2019). Multiobjective evolutionary algorithms: A survey of the state of the art. *Swarm and Evolutionary Computation*, 44, 1–30. <https://doi.org/10.1016/j.swevo.2018.01.004>
7. Khosravi, A., & Piraei, K. (2023). A hybrid NSGA-II and machine learning approach for sustainable energy planning. *Energy Reports*, 9, 450–463. <https://doi.org/10.1016/j.egyrs.2023.01.032>
8. Kidanu, R. A. (2023). Improving multi-objective optimization methods of water distribution systems using improved NSGA-II. *Water*, 15(14), 2561. <https://doi.org/10.3390/w15142561>.
9. Bureau of National Statistics of the Agency for Strategic Planning and Reforms of the Republic of Kazakhstan // [Electronic resource]: URL: <https://stat.gov.kz/ru/region/almatyobl/> . Date of application: 29.08.2025.
10. Information and legal system of regulatory legal acts of the Republic of Kazakhstan. Institute of Legislation and Legal Information, Ministry of Justice of the Republic of Kazakhstan. // [Electronic resource]: URL: <https://adilet.zan.kz/rus/docs/V20D0005841>. Date of application: 1.09.2025.
11. Ahmed, I., & Deb, K. (2022). Reference point based multi-objective optimization for policy making. *Applied Soft Computing*, 128, 109469. <https://doi.org/10.1016/j.asoc.2022.109469>
12. He, Q., & Zhang, Y. (2024). Equity and efficiency in government spending: A computational approach. *Government Information Quarterly*, 41(1), 101830. <https://doi.org/10.1016/j.giq.2023.101830>.
13. Wang, X., & Chen, L. (2023). Fair resource allocation under inequality constraints: An optimization perspective. *European Journal of Operational Research*, 308(2), 623–637. <https://doi.org/10.1016/j.ejor.2022.07.016/>.
14. Jdidou, Y., & Ben Ahmed, S. (2025). Gradient-enhanced evolutionary multi-objective optimization for constrained problems. *Applied Soft Computing*, 151, 111020. <https://doi.org/10.1016/j.asoc.2024.111020>.

NSGA-II ӘДІСІНЕ НЕГІЗДЕЛГЕН ӘДІЛДІК ШАРТТАРЫМЕН ӨНІРЛІК БЮДЖЕТТІ БӨЛҮДІҢ КӨП МАҚСАТТЫ ОПТИМАЛДАУЫ

Аңдатпа. Бұл мақалада ауданаралық теңсіздікті азайта отырып, утилитарлық әл-ауқатты барынша арттыратын аймақтық бюджетті бөлу механизмін құру ұсынылады. Оңтайландырғыш, NSGA-II эволюциялық көп мақсатты алгоритм, қатаң шектеулер арқылы және объективті деңгейде (жан басына шаққандағы Джини индексі азайту арқылы) әділдікті қамтамасыз етеді. Пайдалылықтың аз шығынымен ауданаралық теңсіздікті жою үшін авторлар әділдік шектеулері бар NSGA-II негізіндегі әдіснамасы

берік Парето фронтын жасайтынын және үш жұмыс істейтін балама (тиімділік, теңдік және тізе) ұсынатынын көрсетеді.

Әділдік шектеулері бар NSGA-II негізіндегі әдіснама үш негізгі шешім нұсқасымен тұрақты және түсіндірілетін Парето фронтын жасайтыны көрсетілген: теңдікке бағытталған шешім, тиімділікке бағытталған шешім және ымыраға келу шешімі (тізе нүктесі). Жалпы пайдалылықтың елеусіз жоғалуымен бұл стратегиялар ауданаралық теңсіздікті айтарлықтай азайтуға болатынын көрсетеді. Атап айтқанда, $G(x) \leq \tau$, ең жоғары/мин-қатынас және жан басына шаққандағы ең төменгі деңгей сияқты «қатаң» шектеулерді енгізу реттеуші шекті мәндердің (мысалы, $\tau = 0,21-0,22$) орындалуын қамтамасыз етеді, бұл кішігірім аудандарды алып тастауға жол бермейді. Қаражаттың шоғырлануын азайтудың тағы бір жолы - ойыс пайдалылық функциясын пайдалану.

Кемшілік (тізе) шешімінің салалық профиліне сәйкес, ең жоғары үлестер цифрландыруға (33,49%), көлік транспортына (31,98%) және мәдениетке (24,08%) бөлінді, ал денсаулық сақтау және экологияға сәйкесінше шамамен 0,19% және 0,13% ғана бөлінді. Бұл жағдайда Джини индексі 0,1586-дан (тиімділік) 0,0469-ға (тізе) дейін, яғни 70,44%-ға төмендейді, ал жалпы пайдалылық тек 4,44%-ға төмендейді, бұл тиімділік-теңдік тепе-теңдігін сандық түрде растайды.

Түйін сөздер: NSGA-II, көп мақсатты оңтайландыру, эволюциялық алгоритмдер, әділдік шектеулері, Джини индексі, өңірлік бюджетті бөлу, Парето майданы.

МНОГОЦЕЛЕВАЯ ОПТИМИЗАЦИЯ РАСПРЕДЕЛЕНИЯ РЕГИОНАЛЬНОГО БЮДЖЕТА НА ОСНОВЕ NSGA-II С ОГРАНИЧЕНИЯМИ СПРАВЕДЛИВОСТИ

Аннотация. В данной статье предлагается создать механизм распределения регионального бюджета, который снижает межрайонное неравенство и максимизирует утилитаристское благосостояние. Оптимизатор, эволюционный многокритериальный алгоритм NSGA-II, обеспечивает справедливость посредством жестких ограничений и на уровне целей (минимизируя индекс Джини на душу населения). Чтобы устранить межрайонное неравенство с незначительной потерей полезности, авторы показывают, что их методология, основанная на NSGA-II с ограничениями справедливости, создает прочный фронт Парето и предлагает три работоспособные альтернативы (эффективность, равенство и точка перегиба).

Показано, что методология, основанная на NSGA-II с ограничениями справедливости, создает устойчивый и интерпретируемый фронт Парето с тремя основными вариантами решения: решение, ориентированное на равенство, решение, ориентированное на эффективность, и компромиссное решение (точка перегиба). При незначительной потере совокупной полезности эти стратегии демонстрируют возможность значительного снижения межрайонного неравенства. В частности, включение «жестких» ограничений, таких как $G(x) \leq \tau$, соотношение максимума к минимуму и нижний предел на душу населения, гарантирует соблюдение нормативных порогов (например, $\tau = 0,21-0,22$), что позволяет избежать исключения второстепенных районов. Другой способ снизить концентрацию средств — использовать вогнутую функцию полезности.

Согласно секторному профилю компромиссного решения (колена), наибольшие доли были выделены на цифровизацию (33,49%), транспорт (31,98%) и культуру (24,08%), в то время как здравоохранение и экология получили лишь около 0,19% и 0,13% соответственно. В этом случае индекс Джини снижается с 0,1586 (эффективность) до 0,0469 (колена), т.е. на 70,44%, тогда как общая полезность падает всего на 4,44%, что количественно подтверждает баланс эффективности и равенства.

Ключевые слова: NSGA-II, многоцелевая оптимизация, эволюционные алгоритмы, ограничения справедливости, индекс Джини, распределение регионального бюджета, фронт Парето.

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